Concept Cartoons as a Way to Elicit Understandings and Encourage Reasoning about Decimals in Year 7

Samone Davidson
Monash University
<smdav9@student.monash.edu>

Mike Askew
Monash University
<Mike.Askew@monash.edu>

This paper is located within the research into encouraging learners to reason mathematically and become engaged with concepts rather than just procedures. It reports on a research in progress examining two techniques to be used at the beginning of a sequence of lessons in order to elicit students' prior knowledge and understanding of topics about to be studied. The research will be conducted with two Year 7 classes of girls as they are introduced to units of work on decimals and fractions. Two techniques will be used to elicit the students' prior understandings: Concept Maps and Concept Cartoons. This paper reports on the ideas behind Concept Cartoons and the development of Concept Cartoons for eliciting understandings of decimals. The paper briefly outlines the research into difficulties with decimals before discussing the rationale and design of the cartoons. The paper is concluded with a discussion of how concept cartoons may or may not elicit different understandings to using concept maps.

Introduction

How students are to learn rich and deep decimal concepts is well known to be problematic: assessing learners' prior understandings and building on these is often advocated as one way of addressing this problem. While pre-testing is one means of eliciting prior understanding, pre-tests can potentially fail to uncover misconceptions as students can solve problems using their procedural knowledge rather than displaying a conceptual understanding or the test design may not address particular misconceptions. Concept Cartoons are a means of assessment that may be able to elicit misunderstandings that pre-tests are unable to. While Concept Cartoons have been successfully implemented into the field of science teaching and while research has been conducted on their use in mathematics, there is a need for further research into their design and implementation. This paper reports on research focused on the development of Concept Cartoons as a means of eliciting prior knowledge of decimals and in particular on the design of two Concept Cartoons. This research builds on the insights from science and the research that has been done in mathematics into understanding decimals.

Teaching mathematics is often defined by a rigid curriculum where each topic or concept is given a particular time span. Teachers feel compelled to teach the skills required by the curriculum and find it difficult to explore each concept thoroughly and acknowledge student's misconceptions and prior knowledge. The use of Concept Cartoons would address this issue as they aim to elicit prior mathematical knowledge from the students. The use of Concept Cartoons within the mathematics classroom is expected to engage students in discussing their prior knowledge and encourage them to challenge their existing beliefs, thus enabling misconceptions to be elicited and addressed.

Concept cartoons are designed to be an effective tool to not only gather meaningful information about a student's prior learning in decimals but also to interest and challenge students. The use of cartoons adds a visual stimulus potentially allowing students greater confidence when conveying their ideas in a group situation. Refining, researching and justifying solutions may also ensure that the students have the opportunity to understand a concept from multiple perspectives. Concept cartoons thus enable students to become more

In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia). Singapore: MERGA.

engaged in their learning with greater focus on the concept presented rather than the procedure required to reach a solution.

This paper considers the design of research to be carried out with Year 7 middle school students and discusses the use of Concept Cartoons to encourage mathematical reasoning and effective engagement in the classroom with respect to decimal fractions. The research will explore and compare Concept Cartoons with Concept Mapping as techniques used to access and build on a students' prior knowledge during the introduction of two mathematical topics in Year 7 (fractions and decimals). Two year 7 classes will be involved in the study. In the first topic one class will trial the use of Concept Maps to elicit prior understandings while the other class will use Concept Cartoons. When the second topic commences the two techniques will be reversed. At the completion of the study each class will have accessed and built on their prior knowledge using concept maps and concept cartoons. As this research is yet to be conducted we focus here on the development of Concept Cartoons for decimals.

The paper begins with a brief overview of the research into understanding decimals as this frames the design of the Concept Cartoons. This is followed by a review of the literature into the effectiveness of Concept Cartoons. These two sections are synthesized in an overview of the development of Concept Cartoons and their proposed use in the classroom. Finally we briefly compare the use of Concept Cartoons with Concept Maps.

Understanding Decimals: Key Issues.

Research that has been conducted into decimals provides clues to common misconceptions that exist when learning takes place. This allows the development of Concept Cartoons to focus on areas which students find difficult to gain a conceptual understanding. Focus has been placed on exploring those misconceptions that are often detected.

> 'Numbers are undoubtedly central in mathematics but it is impossible to understand what difficulties children meet with numbers if you do not look at them as magnitudes of different sorts, transformations, or relationships.' (Vergnaud, 1979, p. 265).

Students risk misunderstanding the importance of relationships that exist between whole and decimal numbers as whole number place value is often taught without the extension into decimals. Major principles of whole numbers are not always transferred to the decimal system, but equally some principles that apply to whole numbers can be misapplied to decimals. The research into these misunderstandings provides the basis for the Concept Cartoons. Students who realise the importance of understanding all values in a number will ensure that they understand that decimal numbers are part of something bigger in mathematics.

Misconceptions

In researching the literature, the main misconceptions selected for developing into Concept Cartoons are:

- Longer is larger
- Decimal point ignored
- Misunderstanding zero
- Relating decimals to fractions

Longer is larger. Students may consider that the decimal having the greater number of digits is the larger decimal. For example 2.6498 is larger than 2.65 since it possesses a greater number of digits. This misconception is referred to as the 'longer is larger' strategy (Steinle, 1998b). This reasoning follows the logic that the more digits a decimal possess' the larger it is, despite the position of the decimal point, and is likely to arise from the relationship that for whole numbers longer is indeed larger.

Decimal point ignored. Students may also attempt to solve mathematical problems by ignoring the decimal point. When ignoring the decimal point, students considered 0.5 to be half of 10 (Glasgow, 2000). This also occurred when students were presented with decimals such as 0.3658 and 0.65. Students saw these decimals to be 3658 and 65 respectively, thus they consider 0.3658 to be the larger decimal. Those students who ignored the tenths, hundredth etc. reverted back to their knowledge of whole numbers (Steinle, 1998a). This misunderstanding may also contribute to a student's difficulties in ordering and comparing decimal numbers (Hiebert, 1983).

Misunderstanding zero. Some students find it difficult to assess whether the presence of zero impacts upon the value of the decimal. 0.45 will appear the same as 0.045: once students ignore the zero both decimals appear to be 0.45. On the other hand some students regard equivalent decimals such as 0.40 and .4 to be different values (Hiebert, 1983). A further misconception over zero is in multiplying by 10; students simply added a 0 to the end of the decimal (Hiebert, 1983). Thus 2.65 multiplied by 10 would become 2.650.

Decimals as denominators. The relationship between decimals and fractions causes much confusion for students. When converting between decimals and fractions students may not view a decimal as a part of a whole. Students may recognise 0.4 as $\frac{1}{4}$ and 0.5 as $\frac{1}{5}$. Students may also conclude that 0.4 is bigger than 0.5 since $\frac{1}{4}$ is bigger than $\frac{1}{5}$ (Steinle, 1998a). Evidence by Hiebert suggests that students prior to grade 9 expressed difficulty converting 0.1 to $\frac{1}{10}$ and 0.4 to $\frac{1}{5}$. After this students began to misunderstand the conversion between decimals and fractions confusing $\frac{83}{100}$ for 0.083. (Hiebert, 1983.)

Developing the Concept Cartoons

Why Use Concept Cartoons?

Concept Cartoons have proven effective in creating focused discussions where the presence of misconceptions obtained through previous learning would be uncovered (Kabapinar, 2005). They have been shown to be successful in providing a purpose for scientific investigation by locating concepts in familiar, everyday, situations. This then gives students a familiarity and confidence to explore the situation presented. Concept Cartoons are accessible to the students enabling them to relate to the problem. When students are able to relate to the problem the outcome is seen to be more successful. Whether this extends to mathematics, and decimals in particular, is one of the questions this research will address.

Concept Cartoons aim to elicit a student's prior understanding of the concept and provide an assessment on which to base further learning. Through prior learning, students have already developed an understanding of mathematical ideas. This previous learning

affects their assimilation of new knowledge. Concept cartoons integrate narrative text with dialogue text to form a visual stimulus to challenge student thinking. The dialogue presented in the cartoons creates the opportunity for students to consider alternative ideas relating to the same topic. Concept cartoons may include more than one view that is correct, thus providing the chance to consider which solutions would be more mathematically acceptable.

In the process of debating Concept Cartoons students are expected to investigate misconceptions of the topic, challenge their understanding and explore acceptable knowledge. In Science Education Concept Cartoons have been shown to enhance student motivation, provide a purpose for the learning, minimise classroom management issues and keep students on task (Kabapinar, 2005). The success of Concept cartoons is found in inviting students to engage in the discussions. Each idea contributed to the discussion is worthy of consideration. Thinking is focused as the students find confidence using another persona to voice their opinions.

Keogh (1999) observed the success of Concept Cartoons when researching groups of students using this strategy. She found that students shifted their thoughts on the knowledge, introduced new ideas, realigned their position and found justification to support their ideas. Conflict was generated in the classroom, which was then investigated as students searched to support an idea. When students agreed on an idea as being correct they then needed to work to justify their position and move to develop a more acceptable scientific idea. Although Keogh based her research on the discipline of Science it could be suggested that a similar situation would exist for mathematics. For learning of mathematics using Concept Cartoons, effective use of this strategy must be combined with correct mathematical epistemology and sound teaching practice.

It is encouraging to find that students become more in control of their own learning when using Concept Cartoons. Research conducted by Kabapinar on a group of 11- and 12-year-olds found that students were not pressured into changing their views to those of their peers as students did not see themselves putting forward the ideas but the views of the characters in the Concept cartoons (Kabapinar, 2005). Concept Cartoons provide a challenge for groups to remain open minded about a solution. They also foster the desire to investigate and research to provide justification to prove their solutions. Students' research and motivation provide an effective stimulus for learning in the classroom. From the range of student ideas the challenge emerges for the group to agree and then discover the same mathematical acceptable solution. The promotion of students having a personal involvement in their lesson engages them to strive to find a consensus that can then be justified. Concept Cartoons thus appear able to turn an explanation provided by the teacher into student driven learning.

Using the Concept Cartoons in Class

The success of Concept Cartoons is dependent on how successfully they are implemented into the classroom. If teachers are not enthusiastic when presenting such a strategy the level of success is limited. (Keogh, 1999). Teachers must carefully monitor the interactions between students in each group to ensure that one student does not dominate the discussion; intervention would then be required to ensure the effective use of the Concept Cartoon. Interaction by the teacher is extremely important and questions exploring the reasons behind a student's choice may be an alert for the presence of a misconception. Over time, students become more confident in discussing their own learning and suggesting ideas to their group which could result in further development of the solution. Students who have the same idea may also display a different approach to discuss their understanding.. This reasoning may then form the basis for a future class discussion (Kabapinar, 2005).

Suggestions for the use of Concept Cartoons within the classroom vary in the amount of individual think time, group discussion and teacher interaction. Concept Cartoons could involve students developing their ideas individually before moving to smaller group discussions. Once this is conducted they may then finalise their ideas on a poster for presentation to the class. Students may also seek ideas as a class first, when beginning Concept Cartoons, to allow the teacher a glimpse of any intervention that may be needed at a later date. Presentation of Concept Cartoons may also be done first as a whole class but then students would be given some time to develop their ideas. A small group discussion would then be held allowing students to finalise and justify their ideas. The focus of these smaller groups would be to ensure a team effort is made to investigate all ideas presented and then students would justify their solution.

Concept Cartoon Compared to Concept Maps?

Students access prior understanding when completing both Concept Maps and Concept Cartoons. Concept Maps encourage brainstorming by students who then endeavour to link key ideas and provide a map showing the connectivity of mathematical concepts. They can be continually updated throughout a topic as students gain new knowledge, thus, Concept Maps become a work in progress. Concept Cartoons encourage a focused discussion between students with the task of justifying mathematical knowledge and eliciting misconceptions. They are aimed at discovering the misconceptions in conceptual understanding that can then be worked on in future lessons. Although concept maps create links between prior knowledge they may not reveal conceptual misunderstandings as these may be masked by a student's ability to follow procedures when finding solutions.

Both techniques will address prior knowledge, however, once this is acknowledged a different path is then taken in moving forward to strengthen mathematical concepts and build new understandings. The extent to which each technique is able to elicit misunderstandings is a key aim of the research.

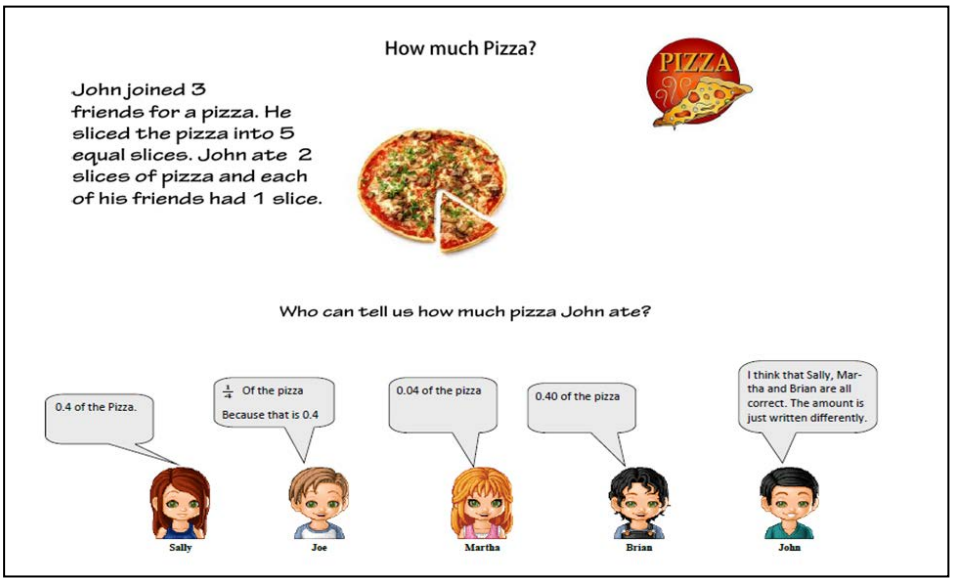
Developing Concept Cartoons for Decimals

Two Concept Cartoons have been developed to address the misconceptions of: longer is larger, decimal point ignored, misunderstanding zero and decimals as denominators.

How Much Pizza?

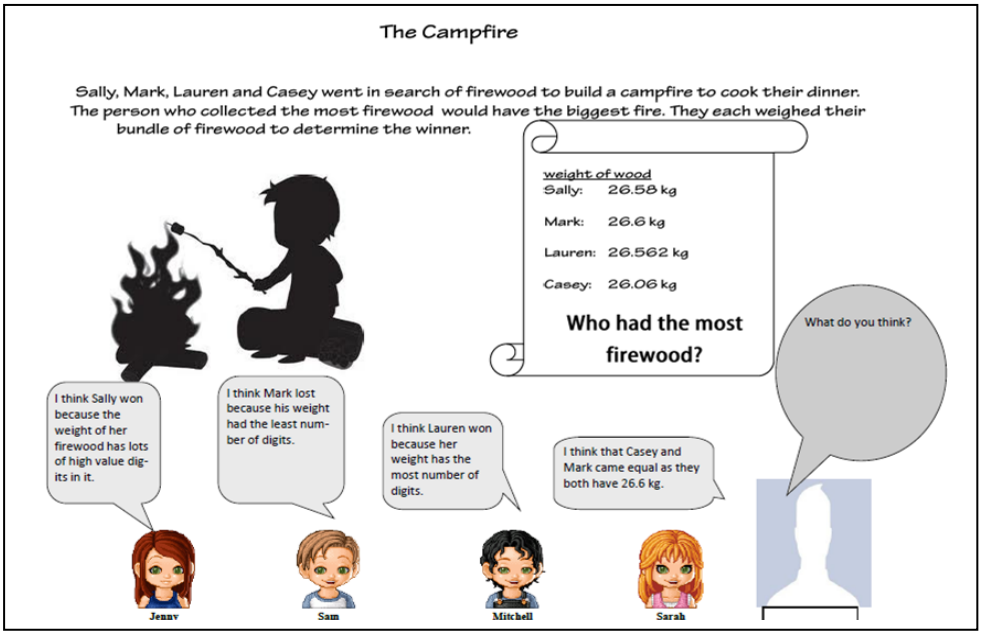
The first Concept Cartoon entitled 'How much Pizza?' explores quantities and decimal representation. Students are asked to choose the correct response when asked how much pizza John ate. Each response is presented as a decimal or fraction. The responses of each of the characters deal with the misconceptions of *misunderstanding zero* and *decimals as denominators*. Sally and Brian have equivalent responses, however, are presented differently with Brian's solution incorporating a zero as the final decimal place value. This highlights a student's understanding of the impact of zero when placed at the end of the number. Martha's response is also focused on the misunderstandings of zero it is placed within a decimal number. Students may ignore the 0 in 0.04 and read this as 0.4. Thus, they may then agree with John.

Joe's response being ¼ highlights the difficulties found when converting decimals to fractions. Students may convert 0.4 to ¼ by placing the decimal placed digits as the denominator in the fraction. Students may then conclude that Sally and Joe have the same chosen correctly.



The Campfire

The second Concept Cartoon explores the ordering of decimals. This cartoon endeavours to elicit the misconceptions of *longer is larger*, *decimal point ignored* and *misunderstanding zero*. In 'The Campfire' students are asked to find the largest decimal number. Decimals of varying lengths are presented.



Classroom Practice

The process of introducing the Concept Cartoons will closely follow the steps below:

1. Give students individual time to choose the idea that they find most mathematically acceptable. Students are to commit to their response of the problem presented.

2. Students will then be placed in a group where they are to go through each idea and choose an appropriate solution. A table (similar to figure 1) will be given to provide the opportunity for the group to sort their ideas.
3. Regrouping will then occur. In the new groups students present their research and justification in the hope of receiving additional feedback.
4. Students join back in their initial groups where they refine, improve (through research) and justify their ideas.
5. Discussion is held by class using the table which students have completed in their groups.

How much Pizza?

Statement	We agree/ Disagree	Proof /Justification
Sally: 0.4 of the pizza		
Joe: ¼ of the pizza because that is 0.4		
Martha: 0.04 of the pizza		
Brian: 0.40 of the pizza		
John: I think that Sally, Martha and Brian are all correct. The amount is just written differently.		

Figure 1. Student Explanation worksheet (adapted from Dabell, 2008, p. 35)

While students are working in groups the teacher is given the opportunity to observe students mathematical reasoning and highlight individual misconceptions which are held. For 'How much Pizza?' a table would take the form of figure 2.

Student Name	Misunderstanding zero	Decimals as denominator	Additional comments

Figure 2. Teacher misconception observations.

Discussion

Decimals are an integral part of learning mathematics and have a place in the world outside the classroom. They assist in describing part of a whole and allow us to express solutions exactly and approximately. The prior learning that students experience impacts how new knowledge is processed by them. Concept Cartoons have been created to assist in eliciting the student's prior knowledge of decimals. With the additional information that could be gained regarding the students learning a more effective program may be adopted.

The opportunity for students to become aware of their own learning is provided through the use of Concept Cartoons and Concept Maps, each of which encourage the eliciting of prior knowledge and misconceptions enabling students to focus on their own understanding. The possible advantage of Concept Maps is that prior knowledge may not have been anticipated by the teacher and the extent of knowledge revealed may be greater or less than expected. A disadvantage however is that the absence of misconceptions being revealed in a Concept Map does not necessarily indicate that a learner's understanding is free of misconceptions. Concept Cartoons have the advantage of directly addressing potential misconceptions, as identified in the research literature, but at the cost of not

enabling learners to reveal the depth or breadth of their understandings. By highlighting and examining these differences in the prior knowledge that each technique elicits it is expected that this research will contribute to helping teachers become aware of the sort of additional support to provide for effective learning to be undertaken.

The effectiveness of using Concept Cartoons to elicit prior knowledge and possible misconceptions in decimals will be the focus of this study. Exploring the most effective way of incorporating these into the classroom practice will assist in gaining greater insight into each students understanding of decimals.

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